Resources for excellence in IIT JEE, Olympiads & NTSE

CHALLENGES IN TRIGONOMETRY

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Foreword

We wanted to present the topics in Plane Trigonometry as a set of Lego blocks – conceptual building blocks, each sitting on top of the other. We decided to create layers of solved examples and problems in between meaningful subsets of concepts. This stratification helped us to string together a set of books, which we hope will help build strong a conceptual foundation for the students of High School Mathematics.

The word trigonometry comes from tri "three" + gonia "angle" + metron "a measure", which is a branch of mathematics that deals with relations between sides and angles of triangles. Therefore trigonometry literally translates to "triangle measurement". The primary application of trigonemetry was in heights and distances - and astronomy. Presumably during the second half of the second century B.C., the first trigonometric table was compiled by the astronomer Hipparchus of Nicaea, who thus earned the right to be known as "the father of trigonometry". Systematic study of trigonometric functions reached India as part of Hellenistic astronomy. In Indian astronomy, the study of trigonometric functions flowered in the Gupta period, especially due to Aryabhata. During the Middle Ages, the study of trigonometry was continued in Islamic mathematics, whence it was adopted as a separate subject in the Latin West beginning in the Renaissance with Regiomontanus. The development of modern trigonometry can be traced to the western Age of Enlightenment, beginning with 17th century mathematics and reaching its modern form with Leonhard Euler.

We sincerely hope that the student is able to get a good grasp of the subject and the techniques after working with the content of this book. If the experience of going through this work is joyful for the student and works as a tool for building his / her understanding, we would be satisfied that we have met the primary objective of this effort.

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Preface

This book is the final book in the set of three dealing with the topic of Plane Trigonometry. It is a companion book to our other volumes "Plane Trigonometry", and "Problems in Plane Trigonometry". As such, it assumes a basic knowledge of, and familiarity with problem solving and Trigonometry.

This is a collection of challenging problems in Plane Trigonometry along with their solutions. These problems are rather more difficult than the tradional text-book problems.

The student is expected to attempt the problems first, before referring to the solutions. In doing so, the depth of understanding in the subject improves. Mathematics is not a spectator sport. It requires patience, perseverance and practice. The level of expertise in the subject in some sense is directly proprotional to the number of problems solved by the student.

The term "solved" is used to imply accuracy of thought, stringing together intermediate steps and accuracy of the final result. In a way, this term refers to the quality of the means and the quality of the end goal for each problem.

This work is a comprehensive self study guide for the students who desire to improve their understanding, appearing for Math related competitive examinations and tests.

There may be situations where the student is stuck and requires a gentle push to make progress. When the student faces such a deadlock, the helping hand comes in the form of the second part of the book, where all the problems are solved completely.

I believe that Astrarka has been blessed to have had the opportunity to work with some of the best and brightest Any work of this magnitude is always a product of teamwork. R Balasubramanian, Shilpa Jaikumar and Venkatratnam Pandit have contributed a great deal to this effort. A big thanks goes to the family members of our team.

They have been a great source of inspiration during this entire effort. They have made a personal sacrifice to ensure that Astrarka succeeds. Without the unflinching commitment and single minded

dedication of my team and the members of their family, this book would have been an exercise in futility.

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Table of Contents

1 Introduction

To say that Trigonometry is useful, therefore, we must learn it, is an understatement. This book serves as the ultimate frontier in your quest for demonstrating your competence in conceptual understanding of the subject, by way of handling 200 odd challenges in Plane Trigonometry.

The student is strongly advised to constantly refer the book "Plane Trigonometry" by the same author, as a ready reckoner until the student is truly confident of recalling the formulas or the steps required to derive them from first principles in case he runs into rough waters.

This book must not be read like a work of fiction. Instead, the student is advised to spend quality time in ensuring conceptual understanding. Solving problems in order to verify our conceptual understanding is extremely important. Most of us believe arriving at the final answer is the ultimate goal. We have come across several books on the subject, where the authors have skipped several steps and simply used the phrase "it follows from the fundamental principles ..." and made a conclusion. We disagree with this approach.

The purpose of the problem solving is build the path to the solution using first principles or well-known formulas - and build an airtight reasoning on how the problem solving process moves towards the final answer. This serves as a demonstration of our understanding of the subject - basics, formulas and methods of manipulation.

This work focuses on the step by step approach to problem solving. We have tried hard to resist the temptation of drawing hasty conclusions, using a broadside "Clearly, it follows", which seems to be a standard fare in most books on the topic. Extreme care has been taken to ensure that the solution involves the concepts covered the chapters leading to the topic under consideration.

All the challenges are outlined in Chapter 3. The student is advised to look at the solutions in Chapter 4, in case he or she requires that gentle push to overcome a minor knot or two.

And like most quiz masters say during the raid fire round – "If you are ready, your clock starts now". Enjoy the ride.

2 Good Habits

There are five fundamental principles, or say good habits that we would like to emphasize before we commence our discussion on Physics.

1. Neatness is conducive to accuracy. Refrain from the temptation to write down something quickly and then scratch the same to make the necessary corrections.
2. One of the weaknesses we find in students while solving word problems is the usage of = sign. This sign has a specific meaning in the world of mathematics. It cannot be used as a way to begin every new line or step in the problem solving process. Use appropriate mathematical signs and symbols. Never use them to mean something vague. = Sign is never a good space filler.
3. Spend a second or two to explain how you arrived at a certain step. Several books and references use a statement, such as ``it follows from the above statement". We have oftentimes wondered how the expression or equation below follows from the one above. A good explanation is an excellent demonstration of your understanding of the underlying principles.
4. When you are faced with several conclusions during a problem solving process, it is a good idea to number the statements or equations. In subsequent steps, you can refer to these conclusions by using the label or the assigned equation number.
5. The easiest of problems attracts the silliest of mistakes. If the problem is easy, motivate yourself to get it right. Do not let over-confidence or carelessness take control of the situation.

3 Problems

1. **Show that if an angle α be divided into two parts, so that the ratio of the tangents of the parts is λ, the difference x between the parts is given by** $\sin x = \frac{\lambda - 1}{\lambda + 1} \sin \alpha$

2. **If** $\tan(\pi \cos\theta) = \cot(\pi \sin\theta)$**, prove that**

$$\cos\left(\theta - \frac{\pi}{4}\right) = \pm \frac{1}{2\sqrt{2}}$$

3. **In any triangle ABC, show that** $\frac{a-b}{c} = \frac{\tan\frac{A}{2} - \tan\frac{B}{2}}{\tan\frac{A}{2} + \tan\frac{B}{2}}$

and $\frac{1 + \tan\frac{A}{2}\tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}}$

4. **An aero plane is travelling east with a constant velocity of 180 km. per hour at a constant height above the ground. At a certain time a man observes it due north of him at an angle of elevation of** $90^\circ 3'$**. At the end of one minute he sees it in a direction** 60° **east of north. At what height is the aero plane travelling, and what is the angle of elevation at which the man sees it in the second observed position?**

5. **If the sides of a triangle are 51, 35 and 26 cm, find the sides of a triangle, on a base of 41 cm, which shall have the same area and perimeter as the first.**

End of Preview.

Rest of the book can be read @
http://kinige.com/kbook.php?id=855

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