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# COORDINATE GEOMETRY

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# Foreword

I have often looked for inspiration from old proverbs and pithy statements. For example, "If you do not know where to go, any road would do." – might be a good way to remember that there can be infinite straight lines passing through a single point. The "where to go" question is answered by the definition of a second point. If you know the starting point and the end point, you can draw one straight line lying on a plane. In general, it is important to know our current position and have a very good idea about where we are headed. If you do not know where you are, no map can help. This book deals with maps, paths and trajectories – in a matter of speaking. Welcome to the world of Coordinate Geometry.

In classical mathematics, analytic geometry, also known **as Coordinate Geometry**, or **Cartesian geometry**, is the study of geometry using a coordinate system and the principles of algebra and analysis. This contrasts with the synthetic approach of Euclidean geometry. In Euclidean Geometry, we defined shapes and concepts – proved the theorem through logical reasoning built on Axioms and other theorems. An axiom denotes primitive statements that are assumed to be true. Theorems are statements that can be proved purely by using axioms and other theorems facts. This is also known as deductive reasoning; and you can see wide applications of this problem solving paradigm – in topics such as symbolic computation and predicate calculus. Coordinate geometry is widely used in physics and engineering, and is the foundation of most modern fields of geometry, including algebraic, differential, discrete, and computational geometry.

Usually the Cartesian coordinate system is applied to manipulate equations for planes, straight lines, and squares, often in two and sometimes in three dimensions of measurement. Geometrically, one studies the Euclidean plane (2 dimensions) and Euclidean space (3 dimensions). We will be looking at 2-dimensional coordinate geometry. As taught in school books, analytic geometry can be explained more simply: it is concerned with defining and representing geometrical shapes in a numerical way and extracting numerical information from shapes' numerical definitions and

representations. The numerical output, however, might also be a vector or a shape.

It is believed that the Greek mathematician Menaechmus solved problems and proved theorems by using a method that had a strong resemblance to the use of coordinates and it has sometimes been maintained that he had introduced analytic geometry. Apollonius of Perga, in On Determinate Section, dealt with problems in a manner that may be called an analytic geometry of one dimension; with the question of finding points on a line that were in a ratio to the others. Apollonius in the Conics further developed a method that is so similar to analytic geometry that his work is sometimes thought to have anticipated the work of Descartes — by some 1800 years. His application of reference lines, a diameter and a tangent is essentially no different than our modern use of a coordinate frame, where the distances measured along the diameter from the point of tangency are the abscissas, and the segments parallel to the tangent and intercepted between the axis and the curve are the ordinates. He further developed relations between the abscissas and the corresponding ordinates that are equivalent to rhetorical equations of curves. However, although Apollonius came close to developing analytic geometry, he did not manage to do so since he did not take into account negative magnitudes and in every case the coordinate system was superimposed upon a given curve a posteriori instead of a priori. That is, equations were determined by curves, but curves were not determined by equations. Coordinates, variables, and equations were subsidiary notions applied to a specific geometric situation. The growth of this field continued until the modern times.

We will be taking a look at the building blocks and concepts – and solve a few problems to give a flavor of problem solving process. A detailed look at problem solving is left to the sequel of this book – "Problems in Coordinate Geometry".

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# Preface

This book is an integral part of a series on Coordinate Geometry.

The book "Coordinate Geometry" covers the concepts involved in the various topics of this subject. A few selected problems are solved after each chapter, to aid the understanding of the student. The book finishes with a collection of problems that the student must practice on, to gain expertise.

"Problems in Coordinate Geometry" is a comprehensive solution set to the battery of over 700 problems in all topics covers in the first volume. The student is expected to make an honest attempt to solve the problems before looking at the suggested solutions. These solutions are systematic and comprehensive. No intermediate steps are skipped; which ensures that the overall flow of the problem solving process starting with the initial conditions to the final solution is maintained.

The best way to use this book is for the student to attempt each problem on his/her own. In doing so, the depth of understanding in the subject improves. Mathematics is not a spectator sport. It requires patience, perseverance and practice. The level of expertise in the subject in some sense is directly proprotional to the number of problems solved by the student. The term "solved" is used to imply accuracy of thought, stringing together intermediate steps and accuracy of the final result. In a way, this term refers to the quality of the means and the quality of the end goal for each problem.

This work is a comprehensive self study guide for the students who desire to improve their understanding, appearing for Mathematics related competitive examinations and tests. These works are based on the gold standard on the topic by SL Loney. They published the book in late 1800s. This forms the central reference in several schools and colleges across the globe.

I believe that Astrarka has been blessed to have had the opportunity to work with some of the best and brightest Any work of this magnitude is always a product of teamwork. R Balasubramanian, Shilpa Jaikumar and Venkatratnam Pandit have contributed a great deal

to this effort. A big thanks goes to the family members of our team. They have been a great source of inspiration during this entire effort. They have made a personal sacrifice to ensure that Astrarka succeeds. Without the unflinching commitment and single minded dedication of my team and the members of their family, this book would have been an exercise in futility.

Chandramouli Mahadevan

# Table of Contents

# 1 Introduction

To say that Coordinate Geometry is useful, therefore, we must learn it, is an understatement. This book focuses on problem solving strategies. We have organized the material into problems, the solution of each problem immediately after the statement. Familiarity with middle school arithmetics and elementary algebra is assumed.

This book must not be read like a work of fiction. Instead, the student is advised to spend quality time in ensuring conceptual understanding. Mathematics requires three skills. Let us look at these issues.

**Comprehension**: At the core of Mathematics, we see the underlying patterns and designs. Each little node in this web is intimately related to the others around it. It is this intricate web of concepts that we need to pay attention to. Expertise and love for the subject is directly related to the quality of our comprehension. Our confidence to deal with issues related to any domain of knowledge is related to the quality of comprehension. So, we need to pay attention to the details. Taking notes is a good way to demonstrate our understanding and reinforce our learnings.

**Problem Solving**: The key to problem solving is practice. Math is not a spectator sport. There are no brownie points for being armchair diplomats. We need to be prepared to jump in and solve the problems that we come across. With practice, and only with practice do we gain the expertise to deploy the right ammunition to crack a problem.

**Goal Clarity**: Solving problems in order to verify our conceptual understanding is extremely important. Most of us believe arriving at the final answer is the ultimate goal. We have come across several books on the subject, where the authors have skipped several steps and simply used the phrase "it follows from the fundamental principles ..." and made a conclusion. We disagree with this approach. The purpose of the problem solving is build the path to the solution using first principles or well-known formulas - and build an airtight reasoning on how the problem solving process moves towards the final answer.

This serves as a demonstration of our understanding of the subject - basics, formulas and methods of manipulation.

# 2. Building Blocks

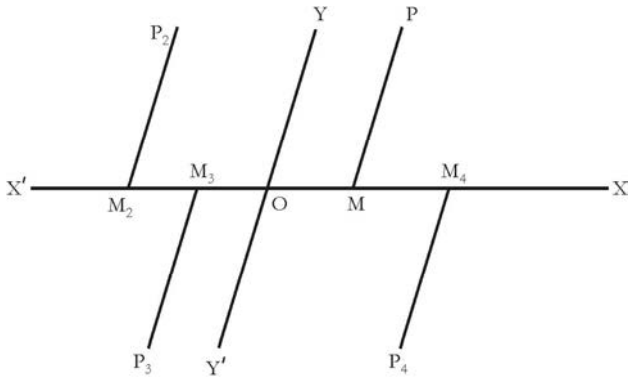


**Fig. 1**

1. Co-ordinates: This is the central theme or concept behind the content of this book. It is important for us to spend a few seconds to understand the term. When we talk of coordinates, we are talking about an unambiguous way to fix the position of an object or a thing with respect to a know reference point, object or a thing. When we say point A is 5 feet away from point B, it could any point which is 5 feet away from point B, in any direction. So, in order to disambiguate, we add the direction to the "left of point A" or "directly above point A" to complete the definition. Armed with this understanding, let us turn our attention to the mathematical notion of coordinates.
2. Let OX and OY be two fixed straight lines in the plane of paper. The line OX is called the X-axis, the line OY is called the Y-axis. Both are called the axes of co-ordinates.
3. The point O is called origin. This serves as the point of reference. Now, we can talk of a point P, which is 5 feet away from O, at an angle of $45^0$ to line OX. This example helps us understand the need for direction to unambiguously define position; in addition to the "distance" from a known reference called "origin".

**End of Preview.**

**Rest of the book can be read @**
**http://kinige.com/kbook.php?id=857**

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