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HIGH SCHOOL ALGEBRA

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Foreword

We wanted to start off the discussion with a short biography of Algebra. We believe that this will help to set the stage to understand the topic more intimately, as opposed to treating this as yet another endurance test with lots of problems to solve and several more symbols and shapes to deal with. We have relied on Wikipedia for creating this sketch[1].

In ancient Egypt and Babylon, people originally learned to solve linear ($ax = b$) and quadratic ($ax^2 + bx = c$) equations, as well as *indeterminate equations* such as $x^2 + y^2 = z^2$, whereby several unknowns are involved. The ancient Babylonians solved arbitrary quadratic equations by essentially the same procedures taught today. It was amazing that they could also handle a few indeterminate equations.

The Alexandrian mathematicians Hero of Alexandria and Diophantus continued the traditions of Egypt and Babylon, but Diophantus's book *Arithmetica* is on a much higher level and gives many surprising solutions to difficult indeterminate equations. So, it is likely that most of us treat the work of Diophantus as the source that dealt with indeterminate equations.

This ancient knowledge of solutions of equations in turn found a home early in the Islamic world, where it was known as the "science of restoration and balancing." The Arabic word for restoration, *al-jabru,* is the root of the word *Algebra.* In the 9th century, the Arab mathematician al-Khwarizmi wrote one of the first Arabic algebras, a systematic exposé of the basic theory of equations, with both examples and proofs.

By the end of the 9th century, the Egyptian mathematician Abu Kamil had stated and proved the basic laws and identities of algebra and solved such complicated problems as finding x, y, and z such that $x + y + z = 10$, $x^2 + y^2 = z^2$, and $xz = y^2$.

1 *Refer to http://en.wikipedia.org/wiki/History_of_algebra*

Ancient civilizations wrote out algebraic expressions using only occasional abbreviations, but by medieval times Islamic mathematicians were able to talk about arbitrarily high powers of the unknown *x,* and work out the basic algebra of polynomials (without yet using modern symbolism). This included the ability to multiply, divide, and find square roots of polynomials as well as knowledge of the binomial theorem. The Persian mathematician, astronomer, and poet Omar Khayyam showed how to express roots of cubic equations by line segments obtained by intersecting conic sections, but he could not find a formula for the roots. A Latin translation of Al-Khwarizmi's *Algebra* appeared in the 12th century. In the early 13th century, the great Italian mathematician Leonardo Fibonacci achieved a close approximation to the solution of the cubic equation $x^3 + 2x^2 + cx = d$. Because Fibonacci had traveled in Islamic lands, he probably used an Arabic method of successive approximations.

An important development in Algebra in the 16th century was the introduction of symbols for the unknown and for algebraic powers and operations. As a result of this development, Book III of *La géometrie* (1637), written by the French philosopher and mathematician René Descartes, looks much like a modern algebra text. Descartes's most significant contribution to mathematics, however, was his discovery of analytic geometry, which reduces the solution of geometric problems to the solution of algebraic ones. His geometry text also contained the essentials of a course on the theory of equations, including his so-called *rule of signs* for counting the number of what Descartes called the "true" (positive) and "false" (negative) roots of an equation. Work continued through the 18th century on the theory of equations, but not until 1799 was the proof published, by the German mathematician Carl Friedrich Gauss, showing that every polynomial equation has at least one root in the complex plane.

The development of varies branches of Algebra continued through the 16^{th} century through to the 19^{th} through contributions from mathematicians in England, Germany, France and Italy.

The essence of the hardwork of our forefathers in Mathematics has helped us to formalize several concepts and tools that we has come to use. I remember an exercise in my school days of trying to explain induction without using any formulas or modern day tools and representation. This exercise taught me to value the work of the Mathematicians. Things that we take for granted have been drilled into our thought process - during our early years at school. Our contribution and our relationship to this field depends upon our ability to internalize these concepts and master the methods of formal problem solving.

Thus, while learning and understanding the concepts and definitions are important, one can build up expertise in Mathematics only by solving problems. Mathematics requires a combination of two skills –comprehension of concepts and related nuances and the ability to recall the appropriate tool, technique, formula or method in madness to solve a problem at hand.

We sincerely hope that the student is able to get a good grasp of the subject and the techniques after working with the content of this book. If the experience of going through this work is joyful for the student and works as a tool for building his / her understanding, we would be satisfied that we have met the primary objective of this effort.

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Preface

This book is an integral part of a 3 volume series on High School Algebra.

The book "High School Algebra" covers the concepts involved in the various topics of this subject. A few selected problems are solved after each chapter, to aid the understanding of the student. The book finishes with a collection of problems that the student must practice on, to gain expertise.

"Problems in High School Algebra" is a comprehensive solution set to the battery of over 1500 problems in all topics covers in the first volume. The student is expected to make an honest attempt to solve the problems before looking at the suggested solutions. These solutions are systematic and comprehensive. No intermediate steps are skipped; which ensures that the overall flow of the problem solving process starting with the initial conditions to the final solution is maintained.

Finally, there are students with an instiable urge for problem solving. "Challenges in High School Algebra" is intended to address this requirement. Over 250 odd gems on the topic have been covered.

The best way to use this book is for the student to attempt each problem on his/her own. In doing so, the depth of understanding in the subject improves. Mathematics is not a spectator sport. It requires patience, perseverance and practice. The level of expertise in the subject in some sense is directly proprotional to the number of problems solved by the student. The term "solved" is used to imply accuracy of thought, stringing together intermediate steps and accuracy of the final result. In a way, this term refers to the quality of the means and the quality of the end goal for each problem.

This work is a comprehensive self study guide for the students who desire to improve their understanding, appearing for Mathematics related competitive examinations and tests. These works are based on the gold standard on the topic by Prof Hall and Prof Knight. They published the book in late 1800s. This forms the central reference in several schools and colleges across the globe.

I believe that Astrarka has been blessed to have had the opportunity to work with some of the best and brightest Any work of this magnitude is always a product of teamwork. R Balasubramanian, Shilpa Jaikumar and Venkatratnam Pandit have contributed a great deal to this effort. A big thanks goes to the family members of our team. They have been a great source of inspiration during this entire effort. They have made a personal sacrifice to ensure that Astrarka succeeds. Without the unflinching commitment and single minded dedication of my team and the members of their family, this book would have been an exercise in futility.

Chandramouli Mahadevan

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1 Good Habits

There are five fundamental principles, or say good habits that we would like to emphasize before we commence our discussion on High School Algebra.

1. Neatness is conducive to accuracy. Refrain from the temptation to write down something quickly and then scratch the same to make the necessary corrections.
2. One of the weaknesses we find in students while solving word problems is the usage of = sign. This sign has a specific meaning in the world of mathematics. It cannot be used as a way to begin every new line or step in the problem solving process. Use appropriate mathematical signs and symbols. Never use them to mean something vague. = Sign is never good space filler.
3. Spend a second or two to explain how you arrived at a certain step. Several books and references use a statement, such as ``it follows from the above statement". We have oftentimes wondered how the expression or equation below follows from the one above. A good explanation is an excellent demonstration of your understanding of the underlying principles.
4. When you are faced with several conclusions during a problem solving process, it is a good idea to number the statements or equations. In subsequent steps, you can refer to these conclusions by using the label or the assigned equation number.
5. The easiest of problems attracts the silliest of mistakes. If the problem is easy, motivate yourself to get it right. Do not let overconfidence or carelessness take control of the situation.

2 Introduction

To say that Algebra is useful, therefore, we must learn it, is an understatement. This book focuses on problem solving strategies. We have organized the material into problems, the solution of each problem immediately after the statement. Familiarity with middle school arithmetics and elementary algebra is assumed.

This book must not be read like a work of fiction. Instead, the student is advised to spend quality time in ensuring conceptual understanding. Mathematics requires three skills. Let us recall these.

Comprehension: At the core of Mathematics, we see the underlying patterns and designs. Each little node in this web is intimately related to the others around it. It is this intricate web of concepts that we need to pay attention to. Expertise and love for the subject is directly related to the quality of our comprehension. Our confidence to deal with issues related to any domain of knowledge is related to the quality of comprehension. So, we need to pay attention to the details. Taking notes is a good way to demonstrate our understanding and reinforce our learnings.

Problem Solving: The key to problem solving is practice. Math is not a spectator sport. There are no brownie points for being armchair diplomats. We need to be prepared to jump in and solve the problems that we come across. With practice and only with practice do we gain the expertise to deploy the right ammunition to crack a problem.

Goal Clarity: Solving problems in order to verify our conceptual understanding is extremely important. Most of us believe arriving at the final answer is the ultimate goal. We have come across several books on the subject, where the authors have skipped several steps and simply used the phrase "it follows from the fundamental principles ..." and made a conclusion. We disagree with this approach. The purpose of the problem solving is build the path to the solution using first principles or well-known formulas - and build an airtight reasoning on how the problem solving process moves towards the final answer. This serves as a demonstration of our understanding of the subject - basics, formulas and methods of manipulation.

We may now proceed to the topics in High School Algebra. Have fun along the way.

End of Preview.

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