PLANE TRIGONOMETRY

CHANDRAMOULI MAHADEVAN

Foreword

We wanted to present the topics in Plane Trigonometry as a set of Lego blocks – conceptual building blocks, each sitting on top of the other. We decided to create layers of solved examples and problems in between meaningful subsets of concepts. This stratification helped us to string together a book, which we hope will help build strong conceptual foundation for the students of High School Mathematics.

The word trigonometry comes from tri "three" + gonia "angle" + metron "a measure", which is a branch of mathematics that deals with relations between sides and angles of triangles. Therefore trigonometry literally translates to "triangle measurement". The primary application of trigonemetry was in heights and distances - and astronomy. Presumably during the second half of the second century B.C., the first trigonometric table was compiled by the astronomer Hipparchus of Nicaea, who thus earned the right to be known as "the father of trigonometry". Systematic study of trigonometric functions reached India as part of Hellenistic astronomy. In Indian astronomy, the study of trigonometric functions flowered in the Gupta period, especially due to Aryabhata. During the Middle Ages, the study of trigonometry was continued in Islamic mathematics, whence it was adopted as a separate subject in the Latin West beginning in the Renaissance with Regiomontanus. The development of modern trigonometry can be traced to the western Age of Enlightenment, beginning with 17th century mathematics and reaching its modern form with Leonhard Euler.

We sincerely hope that the student is able to get a good grasp of the subject and the techniques after working with the content of this book. If the experience of going through this work is joyful for the student and works as a tool for building his / her understanding, we would be satisfied that we have met the primary objective of this effort.

Chandramouli Mahadevan.

Astrarka Educational Solutions Private Limited.

Bangalore, India.

Preface

This work is organized a bit differently from the others. The subject of Plane Trigonometry has been decomposed into a set of concepts. These concepts commence with the notion of an angle and a triangle; and it is on these two concepts the entire domain of Plane Trigonometry stands.

Each concept has been split into a set of axioms and postulates, theorems which require proof or predicates or statements that follow from the knowledge uncovered so far.

After a set of concepts are covered, a set of solved problems are presented which make use of the concepts covered so far. This demonstrates the basic problem solving strategy the student might want to internalize or understand which dealing with problems and challenges in the subject.

The set of solved problems are followed by a set of problems, which the student must try to solve by himself. In doing so, the depth of understanding in the subject improves. Mathematics is not a spectator sport. It requires patience, perseverance and practice. The level of expertise in the subject in some sense is directly proprotional to the number of problems solved by the student. The term "solved" is used to imply accuracy of thought, stringing together intermediate steps and accuracy of the final result. In a way, this term refers to the quality of the means and the quality of the end goal for each problem.

There may be situations where the student is stuck and requires a gentle push to make progress. When the student faces such a deadlock, the helping hand comes in the form of the second part of the book, where all the problems are solved completely.

This work is a comprehensive self study guide for the students who desire to improve their understanding, appearing for Math related competitive examinations and tests.

I believe that Astrarka has been blessed to have had the opportunity to work with some of the best and brightest Any work of this magnitude is always a product of teamwork. R Balasubramanian, Shilpa Jaikumar and Venkatratnam Pandit have contributed a great deal

Table of Contents

1   Introduction

To say that Trigonometry is useful, therefore, we must learn it, is an understatement. This book serves as a conceptual introduction to the subject. It also focuses on problem solving strategies. Any book of Mathematics is incomplete without a bunch of problems to solve. This book is no different. We have organized the material into a sequence of concepts and exercises that make use of those concepts. Some familiarity with algebra, biometry is a prerequisite. Most of the material uses the Pythagorean Theorem. Similarity and congruence of triangles and related theorems and concepts would be extremely handy.

This book must not be read like a work of fiction. Instead, the student is advised to spend quality time in ensuring conceptual understanding. Solving problems in order to verify our conceptual understanding is extremely important. Most of us believe arriving at the final answer is the ultimate goal. We have come across several books on the subject, where the authors have skipped several steps and simply used the phrase "it follows from the fundamental principles ..." and made a conclusion. We disagree with this approach. The purpose of the problem solving is build the path to the solution using first principles or well-known formulas - and build an airtight reasoning on how the problem solving process moves towards the final answer. This serves as a demonstration of our understanding of the subject - basics, formulas and methods of manipulation.

2   Good Habits

There are five fundamental principles, or say good habits that we would like to emphasize before we commence our discussion on Mathematics.

1. Neatness is conducive to accuracy. Refrain from the temptation to write down something quickly and then scratch the same to make the necessary corrections.

2. One of the weaknesses we find in students while solving word problems is the usage of = sign. This sign has a specific meaning in the world of mathematics. It cannot be used as a way to begin every new line or step in the problem solving process. Use appropriate mathematical signs and symbols. Never use them to mean something vague. = Sign is never a good space filler.

3. Spend a second or two to explain how you arrived at a certain step. Several books and references use a statement, such as ``it follows from the above statement''. We have oftentimes wondered how the expression or equation below follows from the one above. A good explanation is an excellent demonstration of your understanding of the underlying principles.

4. When you are faced with several conclusions during a problem solving process, it is a good idea to number the statements or equations. In subsequent steps, you can refer to these conclusions by using the label or the assigned equation number.

5. The easiest of problems attracts the silliest of mistakes. If the problem is easy, motivate yourself to get it right. Do not let overconfidence or carelessness take control of the situation.

3   Measurement of Angles

1. When two line intersect, they form an angle.
2. The two lines are called arms of the angle.
3. The point of intersection is called the vertex.
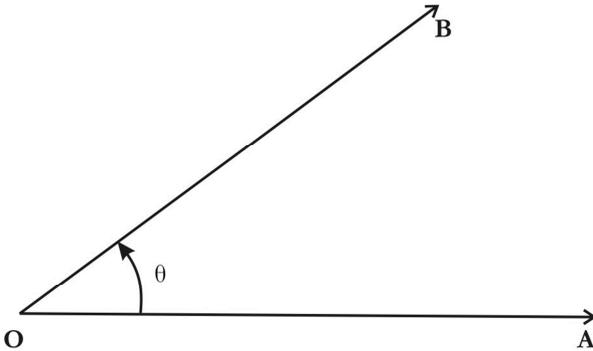
θ : Angle $\overrightarrow{OA}, \overrightarrow{OB}$: Arms of the Angle
O : Vertex

4. We come across various units associated with angles - degrees, radians etc. These are units of measurement of angles.
5. We have several systems of measurement of length, namely - centimeters and meters, feet and miles etc. Similarly, we have three major systems for measurement of angles.
6. They are:
   a. Sexagesimal Measure
   b. Centesimal Measure
   c. Circular Measure
7. In the sections that follow, we will get a better understanding of these three systems of measurement. We will also explore ways of converting angular measurements from one system to another.

3.1 Sexagesimal Measure

1. A right angle is divided into 90 equal parts.
2. Each of these parts consitutes a degree. Therefore, 90 degrees make a right angle.
3. Each degree is further divided into 60 equal parts, each part denotes a minute.
4. Each minute is further divided into 60 equal parts, each part denotes a second.

Unit	equals	is denoted by
1 rightangle	90 degrees	90^0
1 degree	60 minutes	$60'$
1 minute	60 seconds	$60''$

3.2 Centesimal Measure

1. A right angle is divided into 100 equal parts.
2. Each of these parts consitutes a grade. Therefore, 100 degrees make a right angle.
3. Each grade is further divided into 100 equal parts, each part denotes a minute.
4. Each minute is further divided into 100 equal parts, each part denotes a second.

Unit	equals	is denoted by
1 right angle	100 grades	90^g
1 grade	100 minutes	$100'$
1 minute	100 seconds	$100''$

End of Preview.

Rest of the book can be read @

∗ ∗ ∗

Read other books of Chandramouli Mahadevan @